7.5 Integrating Rational Functions by Partial Fractions

Rational Function = ratio of two polynomials

Decomposing a rational function is to change it into the sum of two simpler rational functions that we know how to integrate.

Remember Pre-Calculus or Math-Analysis?

Step 1:

$$\frac{P(x)}{Q(x)} = \frac{A}{first\ factor\ of\ Q(x)} + \frac{B}{next\ factor\ of\ Q(x)} + \cdots$$

Repeat with other factors.

Step 2: fraction busters (multiply each side of the equation by $\frac{\text{the factored form of Q(x)}}{1}$

Step 3: simplify

Step 4: write a system of equations with one equation for the constants only, one only for the terms with x, another only for the terms with x^2 (if there are any), etc.

Step 5: solve for A and B

Step 6: rewrite step 1 with A and B replaced (NOTE: no fractions in your fraction)

Step 7: next class we will then take the integral, remember that $\int \frac{1}{x} dx = \ln|x| + C$

There are other rules if the factors are repeated or if one of the factors is quadratic, but AP does not test on these.

ASSIGNMENT:

Partial Fraction Decomposition **In Exercises 57–70, write the partial fraction decomposition for the rational expression. Check your result algebraically by combining fractions, and check your result graphically by using a graphing utility to graph the rational expression and the partial fractions in the same viewing window.**

57. $\dfrac{1}{x^2 - 1}$

58. $\dfrac{1}{4x^2 - 9}$

59. $\dfrac{1}{x^2 + x}$

60. $\dfrac{3}{x^2 - 3x}$

✓ **61.** $\dfrac{5 - x}{2x^2 + x - 1}$

62. $\dfrac{x - 2}{x^2 + 4x + 3}$

63. $\dfrac{x^2 + 12x + 12}{x^3 - 4x}$

64. $\dfrac{x^2 + 12x - 9}{x^3 - 9x}$

✓ **65.** $\dfrac{4x^2 + 2x - 1}{x^2(x + 1)}$

66. $\dfrac{2x - 3}{(x - 1)^2}$

67. $\dfrac{2x^3 - x^2 + x + 5}{x^2 + 3x + 2}$

68. $\dfrac{x^3 + 2x^2 - x + 1}{x^2 + 3x - 4}$